Homework: Derive the following expression for the derivative of the inverse mapping

$$\left[\frac{\partial}{\partial \Phi}\Phi^{-1}\right]_i =$$

(1) Box-plus and box-minus

$$\boxplus : SO(3) \times \mathbb{R}^3 \rightarrow SO(3),$$
$$\Phi, \boldsymbol{\varphi} \mapsto \exp(\boldsymbol{\varphi}) \circ \Phi,$$
$$\boxminus : SO(3) \times SO(3) \rightarrow \mathbb{R}^3,$$
$$\Phi_1, \Phi_2 \mapsto \log(\Phi_1 \circ \Phi_2^{-1})$$

(2) Rodriguez Formula

$$C(\varphi) = C(\exp(\varphi))$$
$$= I + \frac{\sin(\|\varphi\|\varphi^\times)}{\|\varphi\|} + \frac{(1-\cos(\|\varphi\|))\varphi^{\times^2}}{\|\varphi\|^2}$$
$$C(\varphi) \approx I + \varphi^\times, \quad (\|\varphi\| \approx 0)$$

(3) Inverse Identity

$$\exp(\varphi)^{-1} = \exp(-\varphi)$$

(4) Adjoint Related Identity

$$\exp(\Phi(\boldsymbol{\varphi})) = \Phi \circ \exp(\boldsymbol{\varphi}) \circ \Phi^{-1}$$

(5) anticommutative

$$a \times b = a^\times b$$
$$a \times b = -b^\times a$$

Homework: Derive the following expression for the derivative of the inverse mapping

$$
\begin{aligned}
\left[\frac{\partial}{\partial \Phi}\Phi^{-1}\right]_i &= \lim_{\epsilon \to 0} \frac{(\Phi \boxplus \boldsymbol{e}_i\epsilon)^{-1} \boxminus \Phi^{-1}}{\epsilon} \\
&= \lim_{\epsilon \to 0} \frac{\log(\Phi^{-1} \circ \exp(-\boldsymbol{e}_i\epsilon) \circ \Phi)}{\epsilon} \\
&= \lim_{\epsilon \to 0} \frac{\log(\exp(-\Phi^{-1}(\boldsymbol{e}_i)\epsilon)}{\epsilon} \\
&= -\Phi^{-1}(\boldsymbol{e}_i) = -\boldsymbol{C}(\Phi)^T\boldsymbol{e}_i .
\end{aligned}
$$

$$
\frac{\partial}{\partial \Phi}\Phi^{-1} = -\boldsymbol{C}(\Phi)^T .
$$

Formulate a method to interpolate between two Transformation matrices.

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Consider linear interpolation:

$$x = (1 - \alpha)\, x_1 + \alpha\, x_2, \quad \alpha \in [0, 1]$$

Can we do something similar for SO(3) or SE(3)?

$$(1 - \alpha)\, \mathbf{C}_1 + \alpha\, \mathbf{C}_2$$
$$(1 - \alpha)\, \mathbf{T}_1 + \alpha\, \mathbf{T}_2$$

Formulate a method to interpolate between two Transformation matrices.

Consider linear interpolation:

$$x = (1 - \alpha)\, x_1 + \alpha\, x_2, \quad \alpha \in [0, 1]$$

Can we do something similar for SO(3) or SE(3)?

$$(1 - \alpha)\, \mathbf{C}_1 + \alpha\, \mathbf{C}_2 \notin SO(3)$$
$$(1 - \alpha)\, \mathbf{T}_1 + \alpha\, \mathbf{T}_2 \notin SE(3)$$

Define the following interpolation scheme:

$$\mathbf{C} = \left(\mathbf{C}_2 \mathbf{C}_1^T\right)^{\alpha} \mathbf{C}_1, \quad \alpha \in [0, 1]$$

$$\mathbf{C}, \mathbf{C}_1, \mathbf{C}_2 \in SO(3)$$

Note that:

Define the following interpolation scheme:

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$$\mathbf{C}, \mathbf{C}_1, \mathbf{C}_2 \in SO(3)$$

Note that:

$$\alpha = 0, \text{ we have } \mathbf{C} = \mathbf{C}_1$$

$$\alpha = 1, \text{ we have } \mathbf{C}_2$$

Note that we always have closure with this scheme

$$\mathbf{C} = \left(\mathbf{C}_2\mathbf{C}_1^T\right)^{\alpha}\mathbf{C}_1, \quad \alpha \in [0, 1]$$

Essentially interpolation on the Lie Algebra

$$\mathbf{C}_{21} = \exp\left(\boldsymbol{\phi}_{21}^{\wedge}\right) = \mathbf{C}_2\mathbf{C}_1^T$$

$$\mathbf{C}_{21}^{\alpha} = \exp\left(\boldsymbol{\phi}_{21}^{\wedge}\right)^{\alpha} = \exp\left(\alpha\,\boldsymbol{\phi}_{21}^{\wedge}\right) \in SO(3)$$

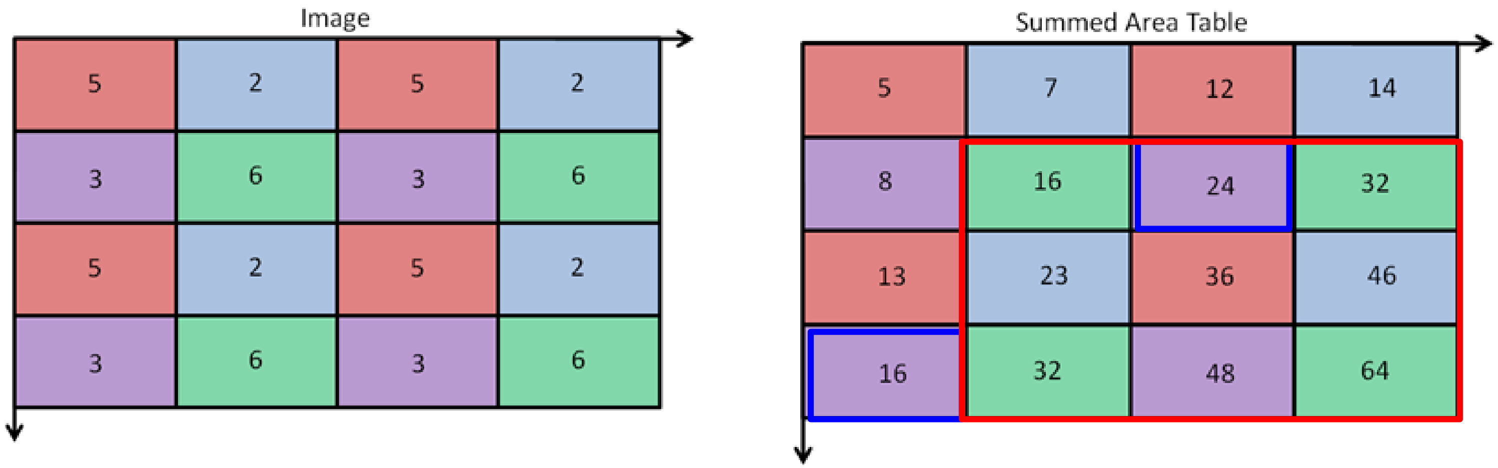
Homework |

Read and Summarize:

1) SURF Detector and Descriptor

1) ORB Detector and Descriptor

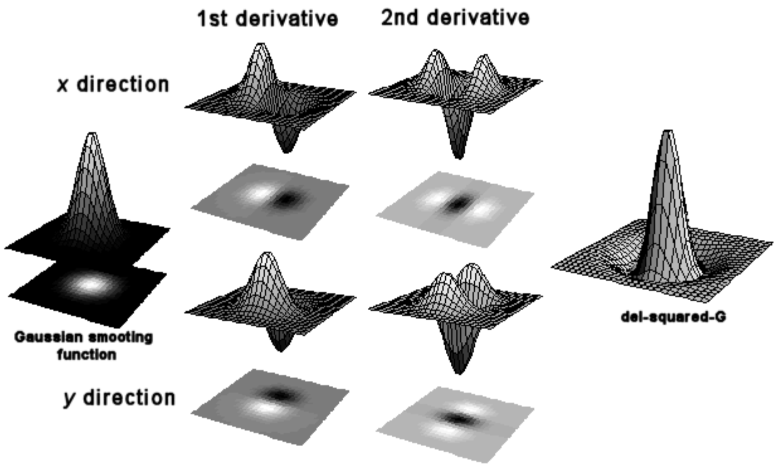
SURF Detector | Integral Image



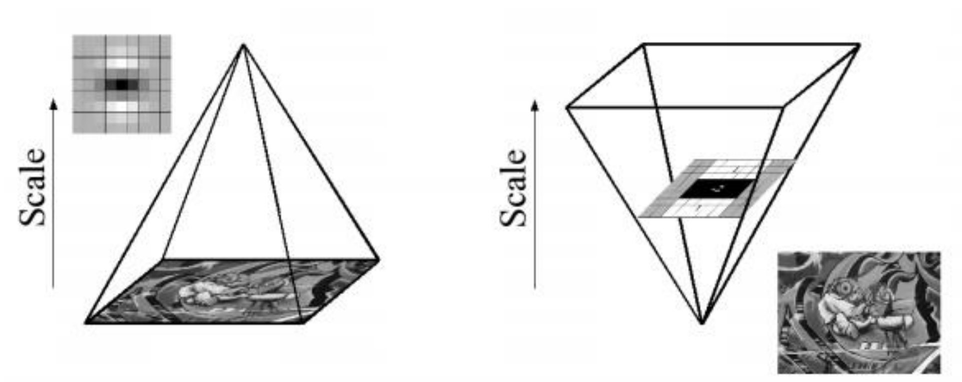
Create: 16 + 12 - 7 + 3 = 24
Create: 16 + 23 - 13 + 6 = 32

Query: 64 + 5 - 14 - 16 = 39
same as 6+3+6+2+5+2+6+3+6 = 39

SURF Detector | Speeded Up Robust Features



Gaussian Partial Derivatives

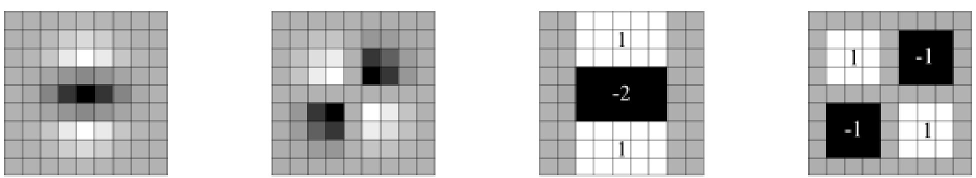


Scale space analysis

$$\mathcal{H}(\mathbf{x},\,\sigma) = \begin{bmatrix} L_{xx}(\mathbf{x},\,\sigma) & L_{xy}(\mathbf{x},\,\sigma) \\ L_{xy}(\mathbf{x},\,\sigma) & L_{yy}(\mathbf{x},\,\sigma) \end{bmatrix}$$

$$\det(\mathcal{H}_{\text{approx}}) = D_{xx}D_{yy} - (wD_{xy})^2.$$

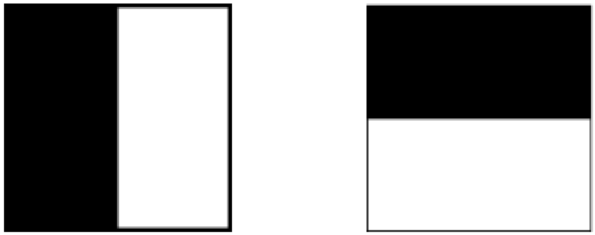
Hessian Matrix and Determinant



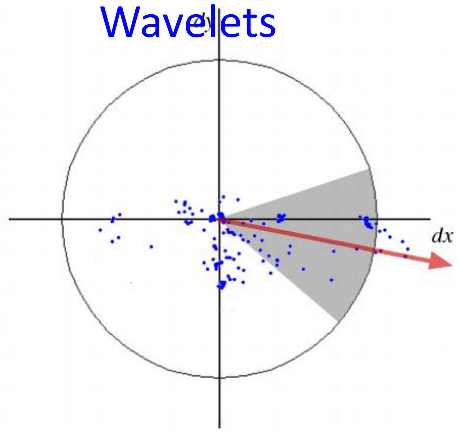
L_{yy} L_{xy} D_{yy} D_{xy}

Box Filter Approximations

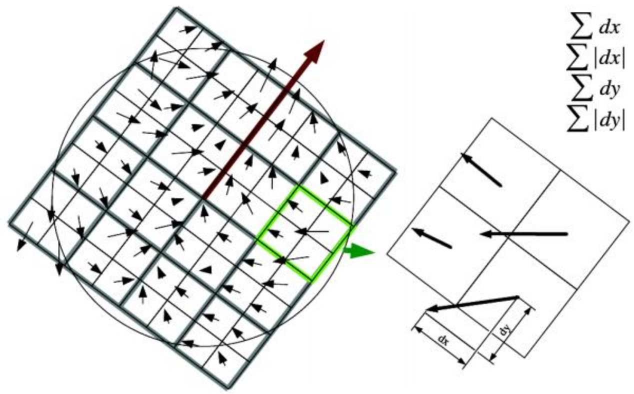
SURF Descriptor |



Haar Wavelets

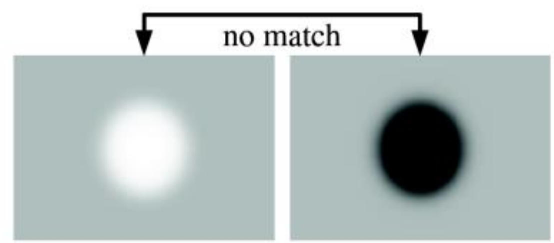


Dominant Orientation



SURF Descriptor

- Each patch is 4 dimensional vector
- 16 Patches give 64 Dimensional Vector



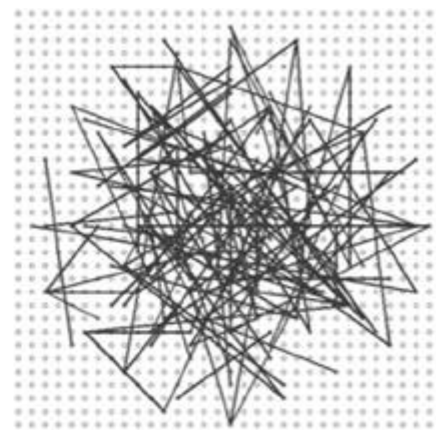
Hessian for black vs blobs

Black vs White blobs

ORB Detector | Oriented fast and Robust Brief

- FAST corner Detector
- Harris Corner Measure
- FAST detected at multiple levels in the Pyramid for Scale Invariance

BRIEF: Binary Robust Independent Elementary Features



- Random Selection of pairs of Intensity Values
- Fixed sampling Pattern of 128, 256 or 512 pairs
- Hamming Distance to compare descriptors (XOR)

ORB Descriptor |

$$m_{pq} = \sum_{x,y} x^p y^q I(x, y)$$

Patch moments

$$C = \left(\frac{m_{10}}{m_{00}}, \frac{m_{01}}{m_{00}} \right)$$

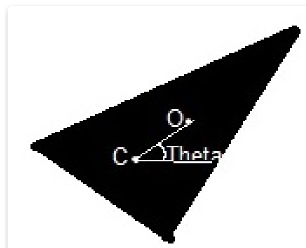
Center of Mass

$$\theta = \text{atan2}(m_{01}, m_{10})$$

Orientation

Learning the pairs

1. Run each test against all training patches.
2. Order the tests by their distance from a mean of 0.5, forming the vector T.
3. Greedy search:
   (a) Put the first test into the result vector R and remove it from T.
   (b) Take the next test from T, and compare it against all tests in R. If its absolute correlation is greater than a threshold, discard it; else add it to R.
   (c) Repeat the previous step until there are 256 tests in R. If there are fewer than 256, raise the threshold and try again.



Angle Calculation

- 300K Keypoints
- 205590 Possible Tests
- 256 dimensional descriptor